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Algorithms to Generate Random Samples following the Swerling Models

Algoritmos para Generar Muestras Aleatorias siguiendo los Modelos de Swerling

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Abstract

This article presents in an easily reproducible way, algorithms to generate samples of the radar video-signal ruled by the four Swerling models, in addition to the case of non-fluctuating target. The two widely used linear and quadratic detectors are taken into account for samples generation and it is verified that these samples present the required statistical characteristics. The proposed algorithms constitute the basis to develop simulations where it is necessary to reproduce video signals prior to any real implementation.

Keywords: Radar, Swerling models, random samples generation.

Resumen

Este artículo presenta de una manera fácilmente reproducible, los algoritmos para generar muestras de la señal de video del radar regidas por los cuatro modelos de Swerling, además del caso de blanco no fluctuante. Los detectores lineal y cuadrático, ampliamente utilizados, son tenidos en cuenta para generar las muestras y se verifica que las mismas presentan las características estadísticas requeridas. Los algoritmos propuestos constituyen la base para desarrollar simulaciones donde sea necesario reproducir señales de video previamente a cualquier implementación real.

Palabras clave: Radar, modelos de Swerling, generación de muestras aleatorias.

Introduction

The Swerling models (Swerling, 1960) were proposed as part of the broad theoretical framework dedicated to modeling the characteristics of the received radar signal and to estimate the statistical behavior of the targets radar cross section (RCS), its temporal variations as well as the detection quality. All the above elements have contributed to use these models as a typical reference to compare the performance of new detection techniques in simulations prior to any implementation. Representative examples include simulations for adaptive detection in Gaussian noise with unknown covariance matrix (Besson et al., 2015), the detection of fluctuating extended targets (Ding et al., 2017), the performance evaluation for ordered-statistic constant false alarm rate detectors (Kong et al., 2016), techniques for small and slow targets detection in marine clutter (Kemkemian et al., 2015), as well as the effectiveness evaluation of chaff recognition algorithms (Bendayan and Garcia, 2015). More recently, some generalized models have been incorporated (Meller, 2018) and even methods based directly on electromagnetic simulation of the RCS (How and Lun, 2016), however the use of these is not yet widespread.

Although several literature describes the Swerling models (Richards, 2005; Richards et al., 2010; Skolnik, 2001), most of them present the detection quality results and make little emphasis in the simulation of the radar video signal ruled by these models. Among the significant examples in this regard, we highlight the works of (Richards, 2008) and (Hughes, 2017), in addition to few functions offered by MATLAB software (MathWorks, 2017). However, the authors have not found explicit procedures and algorithms, especially useful for new researchers interested in the subject. For this reason, the two main objectives of this work are: to propose explicitly procedures that simulate the radar video signal for two types of commonly used detectors (linear and quadratic), as well as to verify that the statistical characteristics of the generated samples follows the Swerling models. In addition to the four fluctuating models, the case of non-fluctuating target is included (Marcum, 1947, 1948) because its simulation is also of interest.

The first section of this article outlines the fundamental characteristics of the Swerling models and the detectors types. Then we propose the algorithms that constitute the core of the present work and proceed to verify that the generated samples have the statistical characteristics required by the models.

Materials and Methods

The Swerling models (Swerling, 1960, 1997) were created taking into account the common problem of detecting M received echoes. The motivation to consider this type of detection is based on the simplified model of a rotating pulse radar, in which the antenna have Ω degrees per second of angular velocity, with θ degrees of half power beamwidth

through the horizontal plane and f_r Hertz of pulse repetition frequency. Although a certain amount of energy is received by the antenna secondary lobes, significant echoes are received only when the target is illuminated by the main lobe, so that for each scan of 360 degrees, a packet of $M = (\theta/\Omega)f_r$ pulses is received.

Each Swerling model is a combination of two probability density functions (pdf) and two decorrelation characteristics for the RCS, resulting in a total of four combinations. Swerling considered two extreme cases for decorrelation characteristics between the M pulses samples. The first case is known as scan-to-scan decorrelation or slow fluctuation, and it assumes that the M pulses amplitudes in each scan (packet) are equals, but the new M pulses amplitudes in the next scan will differ from the previous packet, so that they are independent. The second case is referred to as pulse-to-pulse decorrelation and is applicable to rapidly fluctuating targets, so that the M pulses amplitudes within the packet are independent of each other. Before the advent of the Swerling models, the non-fluctuating target model was proposed by Marcum (Marcum, 1947, 1948), in which the pulses amplitude remains constant, so it does not vary neither between pulses nor between consecutive packets. Some authors refer to this case as Swerling 0 or Swerling 5.

The two pdf used by Swerling to describe the RCS variations are the Exponential and the 4th order Chi-square models. However, to simulate radar signals it is more useful to work directly with the echoes amplitude instead of doing so with the RCS. Since the amplitude is proportional to the square root of the RCS, it follows a Rayleigh pdf (Walck, 2007) and 4th order Chi (Walck, 2007) pdf as shown in Eq. (1) and (2) respectively (Richards et al., 2010), where the average RCS of the target is denoted as $\bar{\sigma}$.

$$p(A) = \frac{2A}{\bar{\sigma}} \exp\left(-\frac{A^2}{\bar{\sigma}}\right) \quad (1)$$

$$p(A) = \frac{8A^3}{\bar{\sigma}^2} \exp\left(-\frac{2A^2}{\bar{\sigma}}\right) \quad (2)$$

The Rayleigh model describes targets composed with several independent reflectors, none of which predominates. For its part, the Chi model describes targets with several reflectors of similar intensity and one that predominates over the others. Table 1 summarizes the characteristics of the models, denoted as Swerling 1, 2, 3 and 4.

Table 1. Characteristics of the Swerling models

Model	Decorrelation	pdf of A	pdf of RCS	Geometry
1	scan-to-scan	Rayleigh	Exponential	Several independent reflectors of similar intensity
2	pulse-to-pulse			
3	scan-to-scan	4 th order Chi	4 th order Chi-square	Several independent reflectors and one predominates
4	pulse-to-pulse			

Detectors Types

The signal received by the radar is typically a narrow band signal, modulated in amplitude, frequency and phase depending on the target characteristics. This means that the echo signal can be written as

$$r(t) = A(t) \sin(2\pi f_0 t + \phi(t)) + n(t) \quad (3)$$

where $A(t)$ represents the envelope, f_0 the carrier frequency (in most cases equal to the intermediate frequency of the receiver), $\phi(t)$ the phase and $n(t)$ is considered additive white gaussian noise (AWGN). The detector extracts the information from the received signal in order to decide if this corresponds to target or noise. In most cases, the detector is located after the intermediate frequency amplifier and its output is known as the video signal.

For the most used detection schemes, the received signal is separated in two channels as shown in figure 1, the upper in phase channel (I) and the lower quadrature (Q) channel, here the (I/Q) detector name (Skolnik, 2001). After mixing $r(t)$ with the local oscillator and the low pass filtering (LPF), the signals $r_I(t) = A(t) \cos \phi(t)$ and $r_Q(t) = A(t) \sin \phi(t)$ are combined to extract the amplitude information as indicated in the figure, depending on the detector type. For the linear detector, the output is directly proportional to the envelope of the received signal, while for the quadratic detector it will be proportional to the square of the aforementioned envelope. The following section proposes the algorithms that allow us to generate the samples corresponding to the video signal for both detector types.

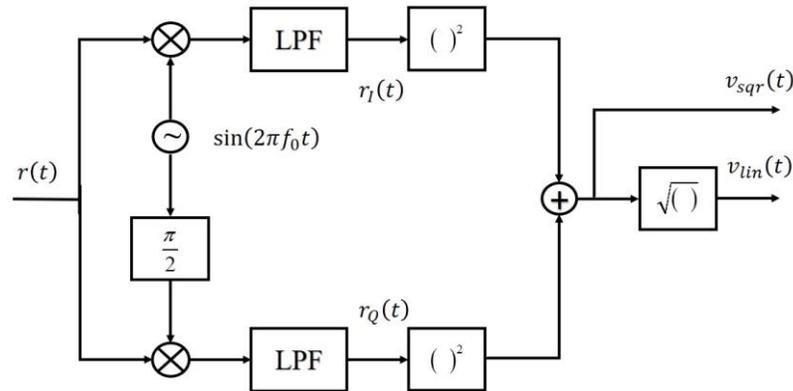


Figure 1. Quadratic and linear detector.

Algorithms to Generate the Random Samples

Figure 2 shows the initial procedure to establish the total number of samples to be generated N , the number of pulses received in each scan M , the Swerling model (SW), the target RCS, the total noise power (N_{pow}) and the detector type

```
phi=random('Uniform', 0, 2*pi, 1, N)
nI=random('Normal', 0, (Npow/2)^0.5, 1, N)
nQ=random('Normal', 0, (Npow/2)^0.5, 1, N)
```

(TD). From these parameters the arrays ϕ , n_I , and n_Q are generated, each one with N samples of the random variables corresponding to the received signal phase (with uniform distribution between 0 and 2π) and to the I and Q channels AWGN, with zero mean and variance $N_{pow}/2$ (equal to the average noise power). Taking as representative example the use of MATLAB (MathWorks, 2017), these three arrays can be generated through the functions

Afterwards, the Swerling model is verified and the algorithm goes to generate the amplitude samples with the corresponding distribution and correlation characteristics.

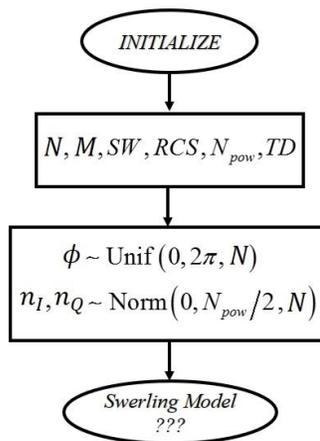


Figure 2. Initial parameters.

If it is desired to simulate a non-fluctuating target ($SW = 0$), the procedure shown in figure 3 is performed. In this case, the amplitude A of the echoes remains constant between consecutive pulses (samples) with a value equal to the square root of the RCS (Richards et al., 2010), while the phase ϕ is the previously generated random variable. Then the noise is added to the echo components s_I and s_Q to create the received signal components r_I and r_Q which will be detected by the corresponding detector in order to obtain the array with the video signal samples.

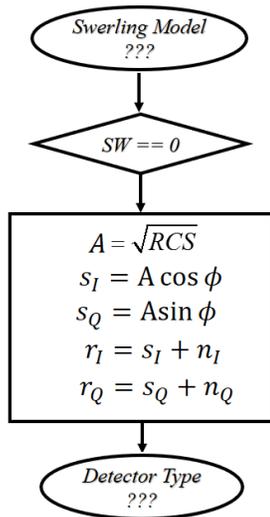


Figure 3. Samples generation for non-fluctuating target.

For the slow fluctuation models of Swerling 1 and 3, the generated samples follow the procedure of figure 4. Depending on the model, the amplitude of the echo will be ruled by the Rayleigh pdf for Swerling 1, or by the 4th order Chi pdf for Swerling 3.

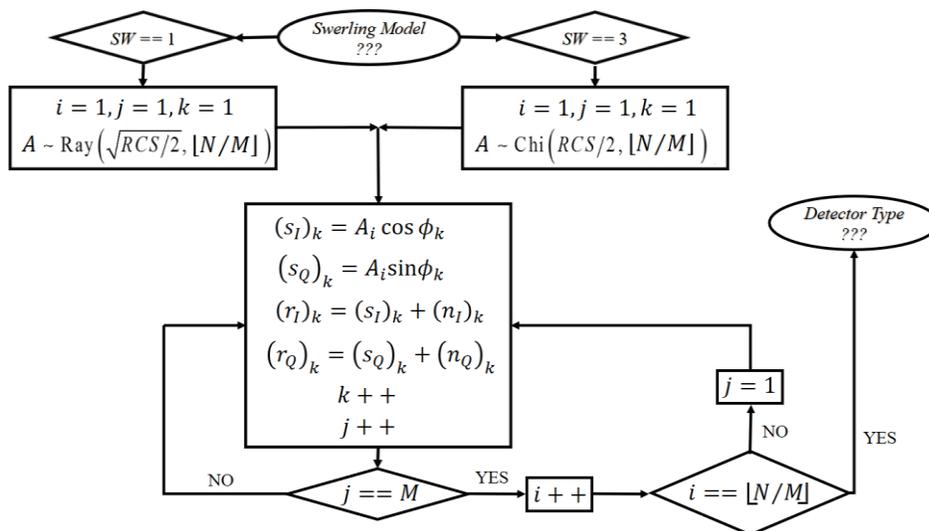


Figure 4. Samples generation for Swerling 1 and Swerling 3 models (slow fluctuation).

Taking MATLAB as example again, the amplitude samples that follow the Swerling 1 model could be generated by the function

```
A=random('Rayleigh', (RCS/2)^0.5, 1, floor(N
```

while that of Swerling 3 could be obtained by means of

```
A=(random('Gamma', 2, RCS/2, 1, floor(N/M))) .^
```

In each case the number of samples is the smallest and closest integer to N/M , with the aim of simulating the packet of equal amplitude for each scan, which constitutes the scan-to-scan decorrelation model or slow fluctuation (Richards, 2008; Richards et al., 2010). For the proposed procedure, the counter i represents the packet index, j indicates if the M pulses of equal amplitude within the packet were generated, while k is used as counter for the samples total.

Similarly, the samples for Swerling 2 and 4 models are generated following the procedure of figure 5. The difference with respect to the previous case is that a different sample is generated for each pulse, which constitutes the pulse-to-pulse decorrelation or fast fluctuation model (Richards, 2008; Richards et al., 2010).

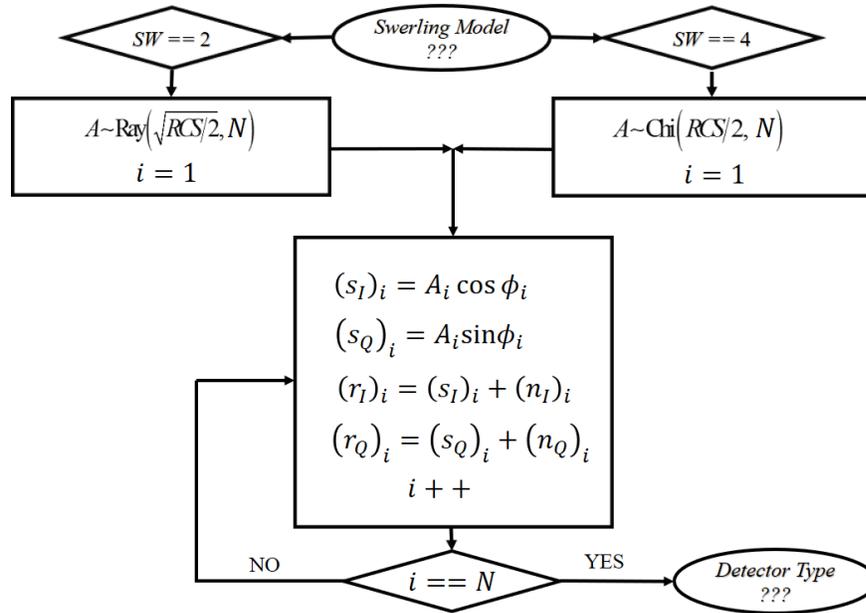


Figure 5. Samples generation for Swerling 2 and Swerling 4 models (fast fluctuation).

From the figure, the meaning of i is clear as index for the individual samples. With MATLAB, the functions that could be used to generate the array of N elements for Swerling 2 could be

```
A=random('Rayleigh', (RCS/2)^0.5, 1, N)
```

whereas in the case of Swerling 4 it could be

```
A=(random('Gamma', 2, RCS/2, 1, N)).^0.5
```

As before, once generated the samples the detector type is determined. Depending on the selected detector type ($TD = 0$ for linear and $TD = 1$ for quadratic) and taking into account the diagram of figure 1, the received signal components are combined as indicated in figure 6 to obtain the video signal v_s . The v_N array contains only noise samples that could be used with other purposes in simulations. An example of the latter would be the DRACEC method (Chávez and Guillén, 2018), where separate noise samples are required to simulate the background and anomaly classes.

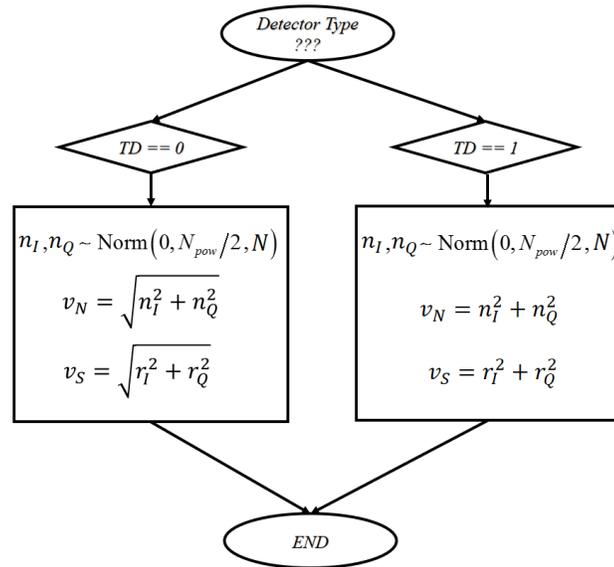


Figure 6. Detector type selection.

Verification of the Samples Statistical Characteristics

Figures 7 and 8 show the estimated pdfs of the pulse amplitude generated with the above mentioned functions through MATLAB (MathWorks, 2017). It is shown in blue the pdfs estimate by histograms, while in red its fit to Eq. (1) and Eq. (2). Besides table 2 shows the fit quality of all the generated random variables taking into account several numeric indicators described below.

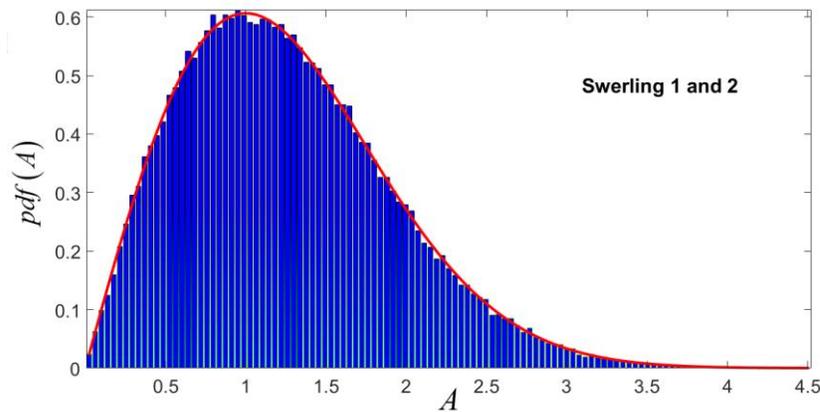


Figure 7. Pulse amplitude pdf estimated by histograms (blue) and its fit (red) to the Rayleigh pdf of Eq. (1) (Swerling 1 and 2).

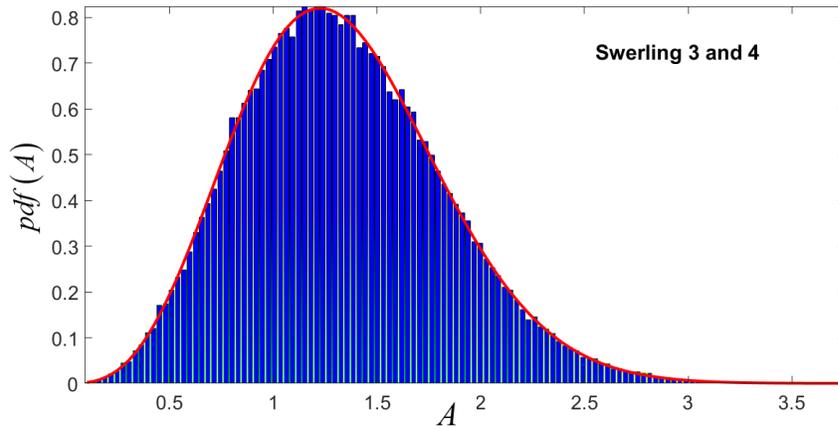


Figure 8. Pulse amplitude pdf estimated by histograms (blue) and its fit (red) to 4th order Chi pdf of Eq. (2) (Swerling 3 and 4).

Table 2. Numerical indicators of the fit quality

Random Variable - pdf	SSE	RMSE	NRMSE	NMSE
ϕ - Uniform	0.0010	0.0041	$-\infty$	$-\infty$
n_I - Gaussian	0.0062	0.0066	0.9662	0.9988
n_Q - Gaussian	0.0053	0.0057	0.9711	0.9991
A - Rayleigh	0.0074	0.0079	0.9687	0.9990
A - 4th order Chi	0.0092	0.0084	0.9714	0.9992

The coefficient SSE is the sum of squared errors of the fit and is defined by Eq. (4) (MathWorks, 2017), so a value close to zero indicates that the estimated pdf is well fitted (see table 2). Henceforward y_i represents the estimated pdf values while \hat{y}_i denotes the fit values.

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (4)$$

The RMSE is the root mean square error of the fit and is determined by Eq. 5 (MathWorks, 2017). A value close to zero indicates a good fit quality (see table 2).

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (5)$$

Finally, the NRMSE and NMSE indicators are calculated using Eq. (6) and Eq. (7) (MathWorks, 2017), which take values between $-\infty$ (bad fit) and unity (perfect fit). The value of these coefficients is not useful to analyze the fit quality

of ϕ to the uniform pdf, because when the fit is a constant, it coincides with its average value \bar{y}_i , causing the denominator of both equations going to zero and therefore both indicators tend to $-\infty$ (see table 2).

$$NRMSE = 1 - \sqrt{\frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{\sum_{i=1}^n (\hat{y}_i - \bar{y}_i)^2}} \quad (6)$$

$$NMSE = 1 - \frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{\sum_{i=1}^n (\hat{y}_i - \bar{y}_i)^2} \quad (7)$$

Conclusions

Through the proposed algorithms it is possible to generate samples of the video signal following the four Swerling models in addition to the non-fluctuating target case. Although the functions to generate the random samples were exemplified using MATLAB, the algorithms implementation is not limited to this tool, since its general nature makes it possible to develop them in any software platform.

On the other hand, the probability densities of the amplitude and phase of the simulated echoes, as well as the AWGN noise taken as interference, were fitted with a high quality to the assumed models. In addition, the provided procedures are easily reproducible, which decreases the development time of the simulations prior to any real implementation, especially for new researchers interested in generating samples of the video signal for the linear and quadratic detectors.

References

- BENDAYAN, M., AND GARCIA, A. (2015). Signal Modeling of Chaff in Naval Environment Simulation. *IEEE Transactions on Aerospace and Electronic Systems*, 51(4), 3161-3166.
- BESSION, O., CHAUMETTE, E., AND VINCENT, F. (2015). Adaptive detection of a Gaussian signal in Gaussian noise. Paper presented at the 6th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), Cancun, Mexico.
- CHÁVEZ, N., AND GUILLÉN, C. (2018). Radar Detection in the Moments Space of the Scattered Signal Parameters. *Digital Signal Processing*, 83 (December), 359-366.
- DING, S., CHEN, D., AND CAO, H. (2017). Performance Analysis of Energy Detector for Fluctuating Extended Targets. Paper presented at the International Conference on Signal Processing, Communications and Computing (ICSPCC), Xiamen, China.

- HOW, T. M., AND LUN, Y. H. (2016). Radar Detector Performance Analysis Using EM Simulations of Targets' RCS. Paper presented at the Radar Conference, Philadelphia, USA.
- HUGHES, E. J. (2017). Piecewise Cumulative Weibull Modelling of Radar Cross Section. Paper presented at the International Conference on Radar Systems, Belfast, UK.
- KEMKEMIAN, S., LUPINSKI, L., NOUVEL, M., CORRETJA, V., AND COTTRON, R. (2015). Slow and Small Target Detection in High Sea States. Paper presented at the Radar Symposium (IRS 2015), Dresden, Ukraine.
- KONG, L., WANG, B., CUI, G., YI, W., AND YANG, X. (2016). Performance Prediction of OS-CFAR for Generalized Swerling-Chi Fluctuating Targets. *IEEE Transactions on Aerospace and Electronic Systems*, 52(1), 492-500.
- MARCUM, J. I. (1947). *A Statistical Theory of Target Detection by Pulsed Radar*: The RAND Corporation, .
- MARCUM, J. I. (1948). *A Statistical. Theory of Target Detection by Pulsed Radar: Mathematical Appendix*: The RAND Corporation, .
- MATHWORKS. (2017). *Matlab Help (Version R2017a)*.
- MELLER, M. (2018). On Bayesian Tracking and Prediction of Radar Cross Section. *IEEE Transactions on Aerospace and Electronic Systems*, Early Access Article.
- RICHARDS, M. A. (2005). *Fundamentals of Radar Signal Processing*. New York, USA: McGraw-Hill.
- RICHARDS, M. A. (2008). *Generating Swerling Random Sequences*. Revised: August 8th, 2017. Available at: https://radarsp.weebly.com/uploads/2/1/4/7/21471216/generating_swerling_random_sequences.pdf
- RICHARDS, M. A., SCHEER, J. A., AND HOLM, W. A. (2010). *Principles of Modern Radar Vol. I: Basic Principles*. Raleigh, NC, USA: SciTech Publishing.
- SKOLNIK, M. I. (2001). *Introduction to Radar Systems (3rd ed.)*. New York, USA: McGraw-Hill.
- SWERLING, P. (1960). Probability of Detection for Fluctuating Targets. *IRE Trans (IT-6)*, 269-308.
- SWERLING, P. (1997). Radar Probability of Detection for Some Additional Fluctuating Target Cases. *IEEE Trans AES-22*, 698-709.
- WALCK, C. (2007). *Hand-book on Statistical Distributions for experimentalists*. Stockholm, Sweden: University of Stockholm.